Design of the Matching Network for the Mexican IPS Array

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1 Introduction :

The Mexican IPS Array design comprises of 16 nos. of full–wavelength dipoles connected in parallel to a two-wire transmission line by a $1\,\lambda$ length of similar trans.line. The trans.line is of characteristic impedance of 413 Ω and the two-wires are un-guarded (i.e. `not` insulated by a dielectric). First measurements of the dipole tuning showed that while all the dipoles tune at 138 MHz (with 1 MHz. deviation), the trans.line at each dipole transformed the tuning to 107 MHz. Hence at the balun end, the dominant 107-109 MHz tuning prevails throughout.

In order to bring back the 138 MHz tuning at the balun port, two solutions exist:

- Estimate the correct length of the $1\,\lambda$ trans.line where the dipole's impedance is simply transformed (no addition or deletion of reactance and hence frequency-independent behaviour of the whole dipole + trans.line ...)
- Incorporate a 'lumped' reactance at the balun to tune-out the un-desired reactances introduced by the $1\,\lambda$ trans.line.

Since the Mexican Array is in advanced stage of construction, the first solution is not cost-effective and demands more man-hours too. The obvious choice is the second one and this note describes the methodology of finding a suitable 'Matching Network' for the array. The network will invariably be a passive one and has to be housed inside the balun–enclosure.

2 Matching Network :

The starting point of designing the network is to get an idea of typical dipole impedance at the interface point of the dipole and $1\,\lambda$ trans.line. Literature yields abundant data based on

measurements, [1] or on computational models. We set out by measuring the dipole impdeances on a Network Analyser.
Typical R and X of dipoles spanning from Row 1 to Row 6 of the array were measured at the four balun-ports. Table 1 gives the measured R and X of the Mexican array.

Table-1

`Network Analyser -- HP 8751A` ; All values are in Ω.

Dipole	Balun-1		Balun-2		Balun-3		Balun-4	
Row	R	X	R	X	R	X	R	X
1	37.8	- 5.7	33.9	-11.3	31.4	-11.6	33.7	- 9.6
2	34.2	- 8.3	33.8	-11.6	27.2	-11.6	36.7	-11.2
3	29.5	- 8.6	31.1	- 4.5	34.4	- 8.0	28.4	-14.6
4	31.9	-13.8	32.1	-11.1	32.8	- 9.3	37.6	- 6.6
5	34.3	-14.0	31.1	- 6.8	26.4	-11.7	34.6	- 8.7
6	22.2	-16.8	28.5	- 6.9	29.1	- 4.6	32.3	-16.5

The average of this 24 sets was taken as the the typical dipole impedance. Deviations from the average do not have significant effect on the Matching Network's parameters, as shown in Appendix–A.
As seen in Table 1, the dipoles plus the trans.line shows capacitive reactance everywhere. An inductance in series could be easily seen as the solution, but it is not so. The network design is decided principally by the location of the R and X of dipole + trans.line on a Smith's chart. This is discussed in length at the Ref.shown [2]. Certain types of networks alone will match the requirement.

2.1 Computation :

Based on Fig.16(c)-[pp.6-15] of the ARRL ref., the network is a series inductance and a shunt-capacitance, for the case of our R and X. Let Z_d be the dipole + trans.line impedance. Since the inductance L_1 is in series with Z_d, let Z_s be,

$$\begin{aligned} Z_s &= Z_d + Z_L \\ &= Z_d + 2\pi f \cdot L_1 \end{aligned} \tag{1}$$

[1] Brown,G.H,Woodward.,O.M,`Experimentally Determined Impedance Characteristics of Cylindrical Antennas`, *Proc. IRE*, Vol.33,1945, pp.257–262.
[2] The ARRL UHF/Microwave Experimenter's Manual- Antennas,Components and Design. Publ.by ARRL,1990

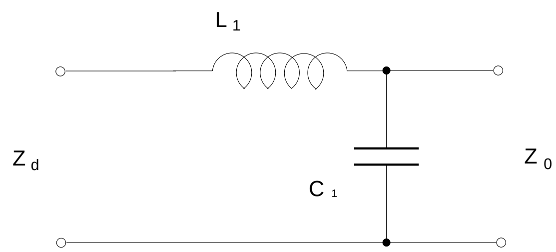

Fig. 1 Matching Network .

As shown in Fig.1, the Z_s and the capacitance are in shunt; the Z_0 is the desired impedance at the balun-port, viz., a normalised impedance of $1.0 + 0j$. By the admittance property,

$$y_s + y_C = y_0 \tag{2}$$

where,

$$\begin{aligned} y_C &= \frac{1}{Z_C} \\ &= \frac{1}{2\pi f \cdot C_1} \end{aligned} \tag{3}$$

If Z_d is expressed as $(R_d - jX_d)$ for this case,(-ve for capacitive reactance) then Z_s becomes,

$$Z_s = R_d + j(t - X_d), \tag{4}$$

where,
t is the reactance of the inductor L_1, viz., $t = 2\pi f \cdot L_1$.

Now y_s becomes,

$$\begin{aligned} y_s &= \frac{1}{R_d + j(t - X_d)} \\ &= \frac{R_d - j(t - X_d)}{{R_d}^2 + (t - X_d)^2} \\ &= \frac{1}{N}\cdot[R_d - j(t - X_d)] \end{aligned} \tag{5}$$

where,

$$N = {R_d}^2 + {X_d}^2 - 2X_d t + t^2 \tag{6}$$

Inserting the relevant expressions in Eq.(2),

$$\frac{1}{N}\cdot[R_d - j(t - X_d)] + \frac{j}{s} = 1.0 + 0j, \tag{7}$$

where s is the reactance of the capacitor C_1, viz.,

$$s = \frac{1}{2\pi f \cdot C_1} \tag{8}$$

Equating the real parts first,

$$N = R_d \tag{9}$$

i.e.,

$${R_d}^2 + {X_d}^2 - 2X_d t + t^2 - R_d = 0 \tag{10}$$

Rearranging the terms,

$$t^2 - 2X_d t + {R_d}^2 + {X_d}^2 - R_d = 0 \tag{11}$$

The above one is a quardratic eqn. in t, and can be solved easily. The two roots of t obtained from this should be examined in conjunction with the following steps :
Equating the imaginary parts next,

$$\frac{(X_d - t)}{N} + \frac{1}{s} = 0 \tag{12}$$

i.e.,

$$\begin{aligned} s(X_d - t) + {R_d}^2 + {X_d}^2 - 2X_d t + t^2 &= 0 \\ t^2 - t(s + 2X_d) + {R_d}^2 + {X_d}^2 + sX_d &= 0 \end{aligned} \tag{13}$$

From Eq.(11),it can be seen that,

$$t^2 - 2X_d t + {R_d}^2 + {X_d}^2 = R_d \tag{14}$$

Hence, the Eq.(13) reduces to

$$\begin{aligned} -st + sX_d + R_d &= 0 \\ s &= \frac{R_d}{t - X_d} \end{aligned} \tag{15}$$

If t has real roots, the above eqn. yields the corresponding s. Putting back the corresponding t and s in the admittance relation, Eq.(2) again, can make the right choice of the solution. Once t and s are known, it is easy to calculate the L_1 and C_1.

In case Eq.(11) does not yield real roots,the computation steps are slightly different: Consider Eq.(13); it is again a quadratic in t and it will have real solutions only when,

$$(s + 2X_d)^2 \;\geq\; 4{\cdot}({R_d}^2 + {X_d}^2 + sX_d) \tag{16}$$

For equality relation, the above quadartic expression in s can be solved and presume a little higher(at the second decimal level...) value for further computation. The following section illustrates the nuances of such selection. Once s is known, t can be solved from Eq.(13).

2.2 Example :

Let $R_d = 0.64$ and $X_d = -0.2$. (`normalized to 50 Ω`). Then Eq.(11) becomes,

$$t^2 - 0.4t - 0.1904 = 0 \tag{17}$$

An examination of the coefficients of the above reveals that the equation does not have any *real roots.*
Next,substituting the R_d,X_d in Eq.(13),

$$t^2 - t(s + 0.4) + 0.4496 + 0.2s = 0 \tag{18}$$

This quadratic in t will have real roots only if,

$$(s + 0.4)^2 \geq (0.8s + 1.7984) \tag{19}$$

or, $s \geq 1.28$.
Let us choose a value of s as 1.3. Then Eqn.(18) becomes

$$t^2 - 1.7t + 0.7096 = 0 \tag{20}$$

Solving, $t = 0.9636$ or $t = 0.73642$ To choose among these two roots, put them back in Eq.(3) along with $s = 1.3$ and check the result :

$$\begin{aligned} y_s + y_C &= \frac{1}{(0.64 - 0.2j + 0.73642j)} + \frac{j}{1.3} \\ &= \frac{(0.64 - 0.53642j)}{0.69735} + 0.7692j \\ &= 0.917765 - 0.7692j + 0.7692j \\ &= 0.917765. \end{aligned}$$

The result is closer to $1.0 + 0j$ for the chosen t; the other root does not satisfy this relation. From t and s, the corresponding inductance and capacitance values are calculated; $L_1 = 42.47$ nH and $C_1 = 17.74$ pF.

The arbitrary selection of s based on the condition shown in Eq.(16) should be closed by finding the $(y_s + y_C)$ or y_0 value; i.e. for the chosen value of s the above expression should be unity. To illustrate this case consider the following R_d and X_d values:
$R_d = 0.846$ and $X_d = 0.038$
Going through the above steps, the condition of s to satisfy,viz., Eq.() yields $s \geq 1.694$. The corresponding t is 0.84397 and the y_0 is 0.6197. Incrementing the s in steps of 0.1 to 0.2 and re-working the computations, yields the following table:

s	t	y_0
1.694	0.84397	0.6197
1.70	0.80574	0.6483
1.90	0.55593	0.8599
2.10	0.46620	0.9410
2.30	0.40916	0.9913

Hence the final choice is the last row of the above Table as y_0 is closer to unity for the given R_d and X_d values.

3 Acknowledgements :

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Appendix –A

Deviations from the average - its' effect on the Matching elements

The computation of the Matching Network was done above for an average value of R_d of 0.64 and X_d of -0.2. A relook at Table–1 with the normalized values shows the following :

Row	Balun-1	Balun-2	Balun-3	Balun4
3	(0.59 - 0.172j)	(0.622 - 0.09j)	(0.688 - 0.16j)	(0.568 - 0.292j)
4	(0.632 - 0.276j)	(0.642 - 0.222j)	(0.656 - 0.186j)	(0.752 - 0.132j)
5	(0.686 - 0.28j)	(0.622 - 0.138j)	(0.528 - 0.234j)	(0.692 - 0.174j)

For example, let us consider two cases,viz., 4th Row – balun 1 and – balun 4. This was chosen since the 4th Row's balun 2 and 3 values are the closest to the average of the ensemble. Balun 1 value of the same row shows a marked deviation in the reactance part alone, while that of balun 4 exhibits a large deviation in the resistive part.

Case - (i):

The values are: $R_d = 0.632$; $X_d =$ - 0.276. Going through Eqns.(13) to (16), and solving the quadratics of s and t, we get $s = 1.27$ and $t = 0.8468$. The corresponding elements are : $C_1 = 18.16$ pF and $L_1 = 48.83$ nH.

Case - (ii):

Here,$R_d = 0.752$; $X_d =$ - 0.132. Again through the same steps of solving s and t, we get $s = 1.51$ and $t = 0.8198$.
and
$C_1 = 15.27$ pF and $L_1 = 47.27$ nH.
Comparison with the elements computed for the average R_d,X_d viz., 17.74 pF and 42.47 nH values, shows that there is no appreciable deviations in them; a single network should suffice to match all the baluns and rows in question, which must be verified experimentally too.

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